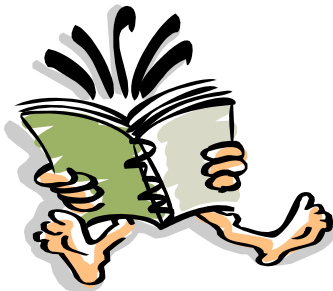


Binary Search



Introduction to Recurrent Algorithms

BINARY-SEARCH

- for an ordered array A , finds if x is in the array $A[\text{lo} \dots \text{hi}]$

Alg.: BINARY-SEARCH ($A, \text{lo}, \text{hi}, x$)

if ($\text{lo} > \text{hi}$)

return FALSE

$\text{mid} \leftarrow \lfloor (\text{lo} + \text{hi}) / 2 \rfloor$

if $x = A[\text{mid}]$

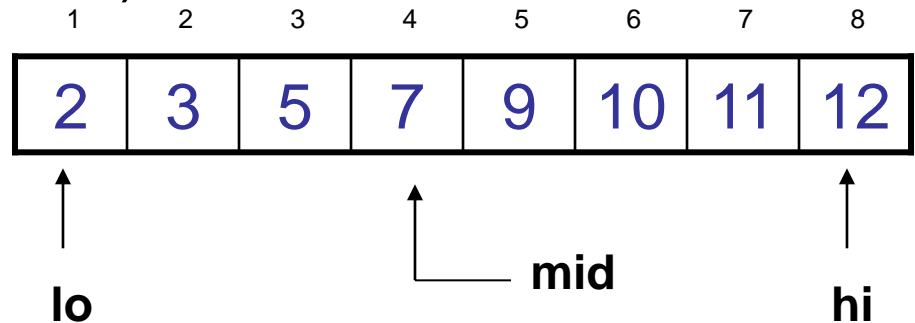
return TRUE

if ($x < A[\text{mid}]$)

 BINARY-SEARCH ($A, \text{lo}, \text{mid}-1, x$)

if ($x > A[\text{mid}]$)

 BINARY-SEARCH ($A, \text{mid}+1, \text{hi}, x$)



Example

- $A[8] = \{1, 2, 3, 4, 5, 7, 9, 11\}$
– lo = 1 hi = 8 $x = 7$

1	2	3	4	5	6	7	8
1	2	3	4	5	7	9	11

mid = 4, lo = 5, hi = 8

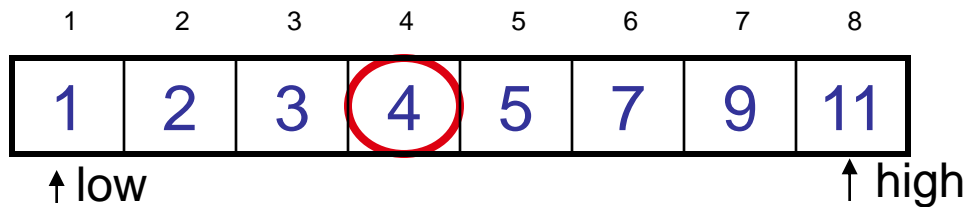
				5	6	7	8
1	2	3	4	5	7	9	11

mid = 6, $A[\text{mid}] = x$
Found!

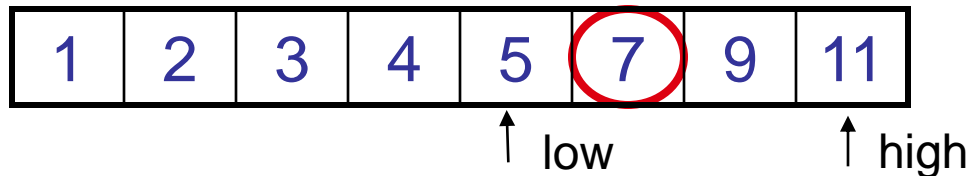
Another Example

- $A[8] = \{1, 2, 3, 4, 5, 7, 9, 11\}$

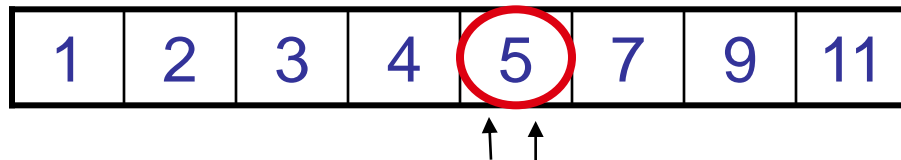
– lo = 1 hi = 8 x = 6



mid = 4, lo = 5, hi = 8

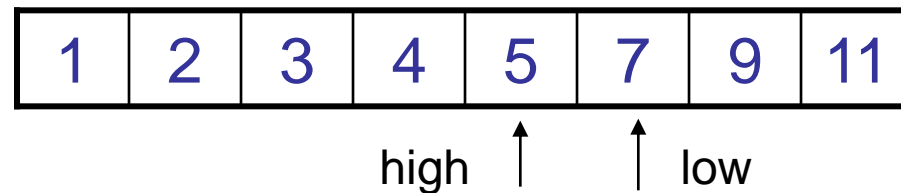


mid = 6, $A[6] = 7$, lo = 5, hi = 5



mid = 5, $A[5] = 5$, lo = 6, hi = 5

NOT FOUND!



Analysis of BINARY-SEARCH

Alg.: BINARY-SEARCH (A, lo, hi, x)

if (lo > hi)

← constant time: c_1

return FALSE

mid ← $\lfloor (lo+hi)/2 \rfloor$

← constant time: c_2

if x = A[mid]

← constant time: c_3

return TRUE

if (x < A[mid])

BINARY-SEARCH (A, lo, mid-1, x) ← same problem of size n/2

if (x > A[mid])

BINARY-SEARCH (A, mid+1, hi, x) ← same problem of size n/2

- $T(n) = c + T(n/2)$

- $T(n)$ – running time for an array of size n

The Iteration Method

- Convert the recurrence into a summation and try to bound it using known series
 - Iterate the recurrence until the initial condition is reached.
 - Use back-substitution to express the recurrence in terms of n and the initial (boundary) condition.

Using Iteration Method

$$T(n) = c + T(n/2)$$

$$T(n) = c + T(n/2)$$

$$= c + c + T(n/4)$$

$$= c + c + c + T(n/8)$$

$$T(n/2) = c + T(n/4)$$

$$T(n/4) = c + T(n/8)$$

Assume $n = 2^k$

$$T(n) = \underbrace{c + c + \dots + c}_{k \text{ times}} + T(1)$$

k times

$$= c \lg n + T(1)$$

$$= \Theta(\lg n)$$

Application

- Number Guessing Game
- Word list

Scope of Research

- Tree Traversal

Assignment

Q.1)What is Binary search?

Q.2)What is best ,worst & Average case analysis of Binary search when search element is not present in the list?

Q.3)Explain Binary search using Divide and conquer method.